NAME: KEY

Rolling Problems

1. A 2 kg disc with a radius of 15 cm is launched in the air so that is rotating about its center mass at 35 rad/s and its center of mass is moving at 7 m/s. What is its total kinetic energy?

$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(2)(7)^2 + \frac{1}{2}\left[\frac{1}{2}(2)(.15)^2\right](35)^2 = \boxed{62.8\ J}$

(I of disc)

2. A 2 kg disc with a radius of 15 cm is rolling without slipping along the ground with a speed of 7 m/s. What is its total kinetic energy?

$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left[\frac{1}{2}mr^2\right]\omega^2$

$\downarrow$ $\swarrow$

$V = r\omega$

$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2 = \frac{3}{4}(2)(7)^2 = \boxed{73.5\ J}$

3. A 20 cm radius ball is rolling without slipping and is rotating at 80 rpm. What is the linear speed of the ball?

$\left(80\ \frac{rev}{min}\right)\left(\frac{1\ min}{60\ s}\right)\left(\frac{2\pi\ rad}{1\ rev}\right)$

$= 8.39\ rad/s$

$V = r\omega$

$= (.2)(8.39)$

$= \boxed{1.68\ m/s}$

4. A 2 kg round shape is rolling without slipping with a speed of 3 m/s. If it has 15 J of kinetic energy, what is the most likely shape of the object?

$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$V = r\omega$

$I = kmr^2$ (k some constant)

$\therefore K = \frac{1}{2}mv^2 + \frac{1}{2}\left[kmr^2\right]\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}kmv^2$

So $15 = \frac{1}{2}(2)(3)^2 + \frac{1}{2}k(2)(3)^2$

$15 = 9 + 9k$

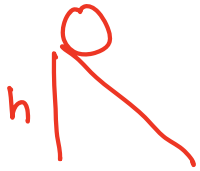
$6 = 9k$

$k = \frac{6}{9} = \frac{2}{3}$

$I = \frac{2}{3}mr^2$

Look @ sheet: $\boxed{\text{Hollow Shell}}$

5. Derive an expression for the speed of a solid uniform sphere of mass M and radius R after rolling down a hill. The sphere starts from a height h, and the hill has a base angle θ. Does your answer depend on the size of the sphere or the angle of the hill?



$\Sigma E_i = \Sigma E_f$

$U_g = K$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ & $V = r\omega$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left[\frac{2}{5}mr^2\right]\omega^2$

$mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$

$mgh = \frac{7}{10}mv^2$

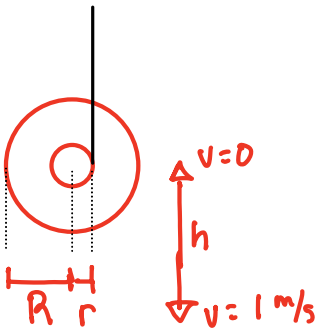
$v^2 = \frac{10}{7}gh$

$\boxed{V = \sqrt{\frac{10}{7}gh}}$

notice there is no "m", "θ" or "r" in the answer - which means that none of those effect the answer.

Rolling Problems

6. A yo-yo can be thought of as being two uniform disks, each of radius 5 cm and mass 150 grams. The string is would around a small post of negligible mass and moment of inertia, but radius 1 cm. Starting from rest, the yo-yo falls a distance h and reaches a final speed of 1 m/s. What was h?



$m = 0.15$ kg
$r = 0.01$ m
$R = 0.05$ m

$\Sigma E_i = \Sigma E_f$

$2(mgh) = 2\left(\frac{1}{2}mv^2\right) + 2\left(\frac{1}{2}I\omega^2\right)$ & $v = r\omega$ ← the little radius!

$2mgh = mv^2 + \left[\frac{1}{2}mR^2\right]\omega^2$ ← big radius

$2mgh = mv^2 + \frac{1}{2}mR^2\left(\frac{v^2}{r^2}\right)$ ← $\omega^2 = \frac{v^2}{r^2}$

$2gh = v^2 + \frac{R^2}{2r^2}v^2$

$h = \frac{v^2}{2g}\left(1 + \frac{R^2}{2r^2}\right)$

$h = \frac{(1)^2}{2(10)}\left(1 + \frac{(.05)^2}{2(.01)^2}\right)$

$h = 0.675$ m